Forward-Link Performance of CDMA Cellular System

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***Abstract*—The impact of the propagation path-loss exponent (γ) on the forward performance of a direct-sequence code-division multiple-access (DS-CDMA) cellular system is investigated. For this purpose, a simple analytical model based on the inverse power-of-distance γ law is developed. The problem of finding proper power-control factors is considered. According to γ, the proper power-control factors are estimated for optimum performance. With these factors, results show that the capacity is reduced by a half by changing γ from 4.5 to 2.5. For this range in γ, power control can approximately double the capacity compared to the case of no power control.**

***Index Terms*—Code-division multiple-access (CDMA) cellular, system performance.**

I. INTRODUCTION

THE direct-sequence code-division multiple-access (DS-CDMA) technique was introduced recently for digital cellular radio communication and has attracted much attention because of its potential to provide higher capacity than that which can be provided by conventional access techniques, such as analog-frequency-division multiple access and digital-time-division multiple access [1], [2]. In a CDMA cellular system, the same band is utilized by all users. The desired signal is extracted from its code, while other signals from system users in the home cell and other cells covering the service area appear as additive interference. The interference received is, therefore, a key factor limiting the radio capacity of the system, that is, the number of simultaneous traffic channels per cell.

Since carrier- and interference-received powers are related to the distance between the mobile unit and cell site, the distribution of the carrier-to-interference (C/I) power ratio in the cell and the number of interfering cells contributing with considerable interference will be affected by the propagation path-loss exponent γ. In the mobile radio environment, γ is always greater than the free-space path-loss exponent $\gamma = 2$ and up to five, depending on the actual conditions of the service area [3]. Some measured values of γ for urban areas are 4.31 in Newark, NJ, 3.05 in Tokyo, Japan [4], and 2.5 in Hamburg, Germany [5].

In the evaluation of the CDMA cellular system performance, the effect of γ has been considered in the analysis for reverse-link mobile-to-base station [6], [7]. Results in [6] show that if γ is reduced by half, the reverse capacity is reduced by up to 50%. For the forward link, most studies use the conventional value of $\gamma = 4$ (the number of interfering cells considered are between 2–18), and the user under consideration is assumed located at the cell corner or moving in a radial direction [1], [6]–[9]. The analysis in [1] and [7] neglects fading, considers shadowing only in [6], and includes the fading channel in [8] and [9]. In this paper, which neglects fading, a simple model is developed for analyzing the performance at a given point in the cell for an assumed γ in the presence of a number of interfering rings of cells.

Forward-link power control is a major issue in CDMA systems. Its purpose is to reduce the interference in neighboring cells by reducing the total transmitted power from cell sites. The scenario is to transmit different powers to different users according to the needs of individual users in the given cell [2]. A distance-driven power-control model was proposed in [1]. In this model, the cell site is assumed to know—in some way—the distance to each user and follows a certain power-control law to control the power transmitted to each of them accordingly. Similar approaches have been used in more elaborate models to analyze the performance of the system, and different methods have been proposed to obtain the power-control factors leading to optimum performance [1], [7]–[9]. A simple method is proposed here to optimize the performance using forward power control.

The aim of this paper is to obtain the proper power control factors for optimum performance and, with these factors, to show the effect of γ on the forward capacity and the distribution of the C/I ratio in the cell.

II. FORWARD CDMA INTERFERENCE

Each cell site is assumed to transmit a total power of S_t. If the power transmitted to a desired user at distance r_o is S_o, then the interference received by this user I_o from its cell site is $\alpha(S_t - S_o)r_o^{-\gamma}$, where α is a constant. Since $S_o << S_t$ in a CDMA system, I_o is approximately

$$I_o(r_o) = \alpha S_t r_o^{-\gamma}. \tag{1}$$

In a multicell system, interference is received by users from the surrounding cell sites. It becomes more significant in a mobile radio environment with low γ. To carry out the interference analysis, the service area is modeled by a lattice of hexagonal cells, each of radius R and having a base station at the center. For any particular cell, ring 1 will refer to the cells adjacent to it, ring 2 will refer to those around and adjacent to ring 1, etc. In Fig. 1, the interfering cells are divided by the bold lines into three symmetric sectors, i.e., we can always find three cells, one from each sector, that have the same distances from the center and are 120° from each other. Using the normalized i–j axis of sector 1, the normalized distance between a cell site having the coordinates (u, v) and the home cell site is

$$d = \sqrt{u^2 + v^2 - uv}.$$

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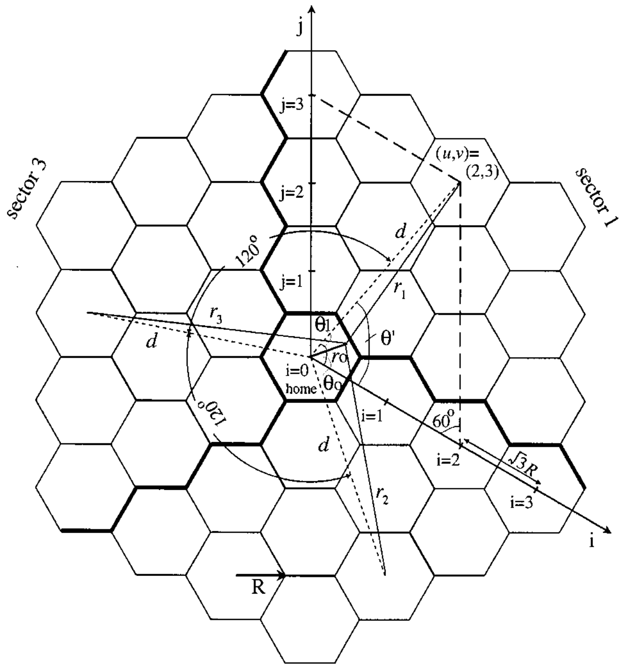

Fig. 1. CDMA interference calculation geometry.

For a user at location (r_o, θ_o), its distance r_1 from the cell site (u,v) can be found from d, r_o, and the angle between them (θ_1). Similarly, its distances r_2 and r_3 from the other two symmetric cell sites can be found from d, r_o, and the corresponding angles θ_2 and θ_3 for sectors 2 and 3, respectively. With reference to the geometry shown in Fig. 1, it is desirable to express the interference I_J received by a user in the home cell from all cell sites in rings up to the Jth around the home cell. For the user at location (r_o, θ_o), it can be shown that I_J is given by

$$\begin{aligned} I_J(r_o,\theta_o) = \sum_{j=1}^{J} \Bigg\{ & \sum_{i=0}^{j} \alpha S_t \left[r_1^{-\gamma}(u,v) + r_2^{-\gamma}(u,v) \right. \\ & \left. + r_3^{-\gamma}(u,v) \right]_{\substack{u=i \\ v=j}} + \sum_{i=1,j>1}^{j-1} \alpha S_t \left[r_1^{-\gamma}(u,v) \right. \\ & \left. + r_2^{-\gamma}(u,v) + r_3^{-\gamma}(u,v) \right]_{\substack{u=j \\ v=i}} \Bigg\} \end{aligned} \tag{2}$$

where

$$r_n(u,v) = \sqrt{r_o^2 + 3d^2R^2 - 2\sqrt{3}r_o dR\cos(\theta_n)}$$

$n = 1, 2,$ and 3 and

$$\theta = \cos^{-1}\left(\frac{2u-v}{2d}\right)$$

$$\theta_1 = |\theta - \theta_o|$$

$$\theta_2 = \frac{2\pi}{3} - (\theta - \theta_o)$$

$$\theta_3 = \frac{2\pi}{3} + (\theta - \theta_o)$$

are used to simplify the notation. Because of using the symmetry mentioned above, the summation limits in (2) apply for the cells in sector 1 only. For example, when $j = 3$, ring 3, the cells (0,3), (1,3), (2,3), (3,3), and (3,1) and (3,2) are covered by the first and the second inner summation terms, respectively. In the following numerical evaluation, θ_o will be taken as 30°, except where otherwise specified. It will be assumed that $J = 10$ for $\gamma \geq 3$, and $J = 15$ for $\gamma = 2.5$ is a sufficient number of interfering rings. The normalized interference given by $I_{Jn} = I_J/S_t$ will be used in the following section.

To evaluate the effect of γ and power control on the distribution of the C/I in the cell, a performance factor will be user defined by

$$G(r,\theta) \triangleq N\left(\frac{C(r)}{I(r,\theta)}\right) \tag{3}$$

where

r and θ polar coordinates in the home cell;
N capacity.

For a particular value of N users/cell, G is determined by the distribution of the C/I in the cell. Alternatively, for all users receiving the required C/I, $(C/I)_{\mathrm{req}}$ or, better, the achievable capacity N_a can be found from

$$N_a = \frac{G_{\min}}{\left(\frac{C}{I}\right)_{\mathrm{req}}} \tag{4}$$

where $G_{\min}$ is the minimum of (3). The optimum distribution of G is achieved for $G_{\min}$ that leads to the highest N_a and with minimum variation of G, i.e., with minimum difference between the maximum and minimum of G. The capacity given by (4) does not include the effects of voice activation or cell-sectorization-enhancing factors.

III. Forward-Link Power Control

It is assumed that N CDMA channels are simultaneously activated by N users distributed in a cell of radius R with density of users

$$\rho = \sqrt{2N/3\sqrt{3}R^2}.$$

All cell sites are assumed to follow an identical power-control law. The power-control law used here is similar to that proposed in [1], where the transmitted power from a cell site to a user at distance r is assumed proportional to r^c, where c is a number. It can be expressed by

$$S_1(r) = S_R\left(\frac{r}{R}\right)^c \tag{5}$$

where S_R is the power required for a user at the cell corner. Assuming a high density of users, the total power S_{t1} transmitted from a cell site can be found from (5) by integrating the power transmitted from the cell site to the users in an element of area, $dA = r\,dr\,d\theta$, over a disk of radius R_d that has the same area as a hexagon cell, i.e.,

$$R_d = \sqrt{3\sqrt{3}/2\pi}R.$$

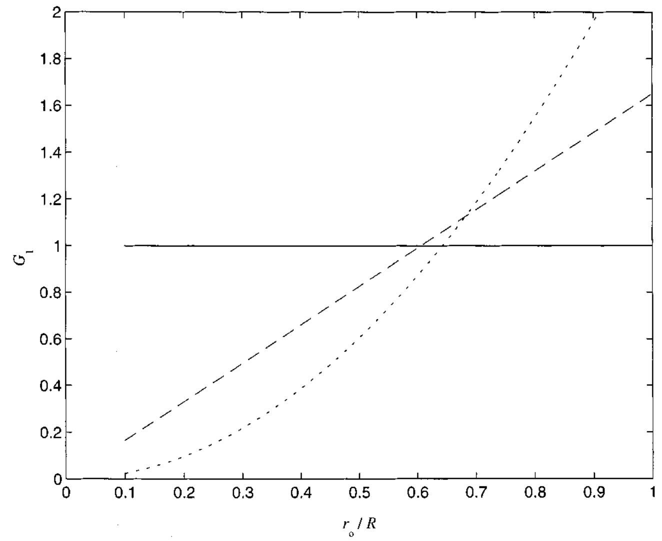

Fig. 2. Performance factor G_1 of single-cell system. $c = 0$ (solid line), $c = 1$ (dashed line), and $c = 3$ (dotted line).

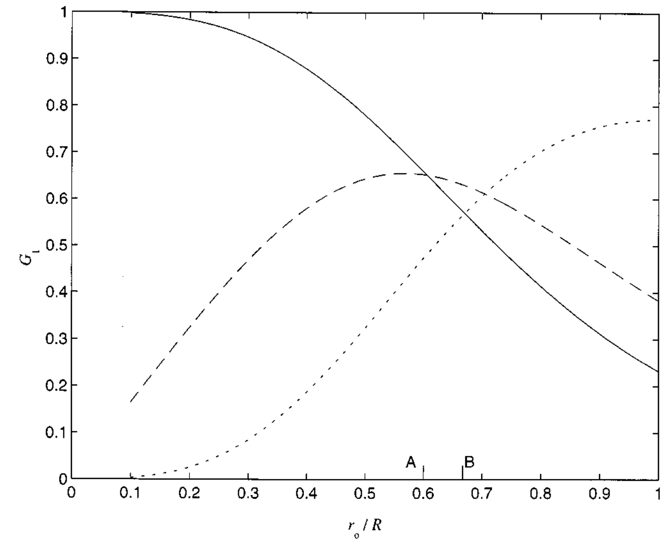

Fig. 3. Performance factor G_1 of multicell system $\gamma = 3$. $c = 0$ (solid line), $c = 1$ (dashed line), and $c = 3$ (dotted line).

Thus

$$S_{t1} = \int_0^{2\pi} \int_0^{R_d} S_1(r) \rho r \, dr \, d\theta = S_R N \left(\frac{2}{c+2} \right) \left(\frac{R_d}{R} \right)^c. \tag{6}$$

Ignoring background noise compared to CDMA interference, the C/I at the desired user location (r_o, θ_o) is found from

$$\frac{C(r_0)}{I(r_o, \theta_o)} = \frac{\alpha S_1(r_o) r_o^{-\gamma}}{I_o + I_J(r_o, \theta_o)}. \tag{7}$$

From (1), (5), and (7), the performance factor defined in (3) can be written

$$G_1(r_o, \theta_o) = \frac{1}{\left(\frac{2}{c+2} \right) \left(\frac{R_d}{r_o} \right)^c \left(1 + \frac{I_{Jn}(r_o, \theta_o)}{r_o^{-\gamma}} \right)}. \tag{8}$$

In a single-cell system, $I_{Jn} = 0$ in (8) and G_1 becomes independent of γ. The best distribution of G_1 is then obtained when $c = 0$, no power control, compared to the cases with power control applied, for example, $c = 1$ and $c = 3$ in Fig. 2. For a multicell system, by substituting (2) into (8), G_1 is found for $c = 0, 1$, and 3 and with $\gamma = 3$, as shown in Fig. 3. With $c = 0$, the minimum point of G_1 that determines the capacity is equal to 0.2321. This is much less than unity achieved in a single cell system. For $\gamma = 2.5, 3.5, 4$, and 4.5, the *minima* of G_1 without power control are found to equal 0.1729, 0.2687, 0.2922, and 0.3092, respectively.

It can be observed from Fig. 3 that power control improves G_1 near the cell boundary, but degrades it near the cell site. Such degradation can be compensated by using adjusted power control as proposed in [1]. Power control is applied only for users located at a distance greater than a threshold distance from the cell site and by transmitting constant power to the other users. Two power-control factors need to be found—the threshold distance r_p and the power-control exponent c. The factor r_p was found in [1] assuming that only home interference is received at this point. In [7], empirical values of r_p were investigated, and in [8], statistical criteria were proposed. We have found r_p to analytically suit the model with a large number of interfering cell sites. Then, a proper value of c is determined.

For a tradeoff distribution of the performance factor between the cases of transmission with power control and without power control, the distance from the cell site at which the condition

$$G_1|_{c=0} = G_1|_{c \neq 0} \tag{9}$$

is satisfied determines the point r_p where power control improves the performance factor for users in the region $r > r_p$ compared to the performance factor without power control, for example, points A and B in Fig. 3. Thus, at $r_o = r_p$, (8) and (9) give

$$r_p = R_d \left(\frac{2}{c+2} \right)^{1/c}. \tag{10}$$

With adjusted power control, the power transmitted to users within $r \leq r_p$ is equal to the minimum power S_p required for a user at distance r_p. The transmitted power from a cell site with adjusted power control can be expressed by

$$S_2(r) = \begin{cases} S_p & 0 < r \leq r_p \\ S_p \left(\dfrac{r}{r_p} \right)^c & r_p < r \leq R \end{cases}. \tag{11}$$

The total power transmitted by cell site can be found from

$$\begin{aligned} S_{t2} &= \int_0^{2\pi} \int_0^{r_p} S_p \rho r \, dr \, d\theta + \int_0^{2\pi} \int_{r_p}^{R_d} S_p \left(\frac{r}{r_p} \right)^c \rho r \, dr \, d\theta \\ &= \frac{2\pi S_p N}{3\sqrt{3} R^2} \left[r_p^2 + \left(\frac{1}{r_p} \right)^c \left(\frac{2}{c+2} \right) \left(R_d^{c+2} - r_p^{c+2} \right) \right]. \end{aligned} \tag{12}$$

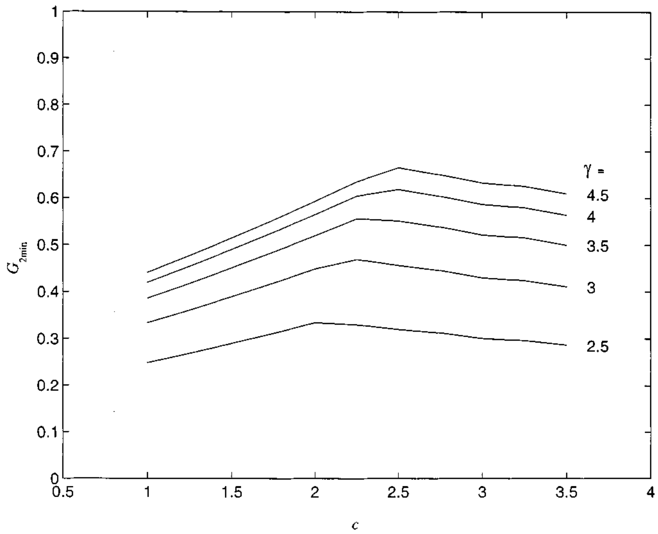

Fig. 4. Minimum of the performance factor G_2 versus the power-control exponent c.

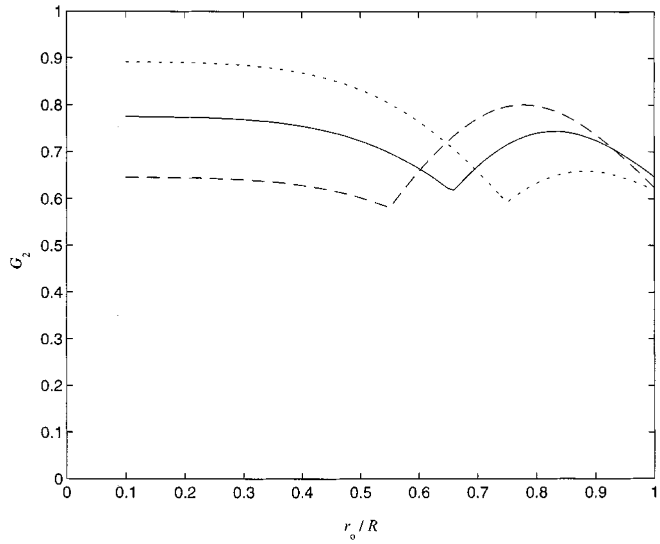

Fig. 5. c, $r_p = 2.5, 0.657R$ (solid line), c, $r_p = 2, 0.55R$ (dashed line), and c, $r_p = 3, 0.75R$ (dotted line).

Following the same procedure used to get (8), it can be shown, using (11) and (12), that the performance factor with adjusted power control G_2 is given by

$$G_2(r_o, \theta_o) = \begin{cases} \dfrac{1}{f(c)\left(1+\dfrac{I_{Jn}(r_o,\theta_o)}{r_o^{-\gamma}}\right)} & 0 < r_o \le r_p \\ \dfrac{1}{f(c)\left(1+\dfrac{I_{Jn}(r_o,\theta_o)}{r_o^{-\gamma}}\right)\left(\dfrac{r_p}{r_o}\right)^c} & r_p < r_o \le R \end{cases} \tag{13}$$

where

$$f(c) = \left(\frac{r_p}{R_d}\right)^2 + \left(\frac{2}{c+2}\right)\left(\frac{R_d^{c+2} - r_p^{c+2}}{R_d^2 r_p^c}\right).$$

It is clear from (13) that c highly affects the distribution of G_2. The effort now is to determine the value of c leading to optimum performance.

IV. Forward-Link Performance

To determine the proper power-control factors for optimum performance, the minimum of (13), $G_{2\min}$, is plotted in Fig. 4 as a function of c with r_p, as given by (10) for different values of γ. The proper value of c is determined from the maximum point of each curve in Fig. 4. For example, the proper values of c for $\gamma = 4$ equal 2.5, as can be seen from Fig. 4, then $r_p = 0.657R$ from (10). The corresponding values of c, r_p reported in [1] and [7] are 2, 0.55R and 3, 0.75R, respectively. Using these three values of r_p and c, G_2 is plotted in Fig. 5. It is seen that the first factor achieves more uniform distribution of C/I than the others with almost the same or higher capacity.

Fig. 4 indicates that N_a is more sensitive for the variation in c below the proper value than for its variation above the proper value. The maximum points of $G_{2\min}$ in Fig. 4 are approximately double the corresponding values without power control given in the previous section, which means that the capacity is doubled using the power control. From the results shown in Fig. 4, when γ is changed from the highest to the lowest assumed values, 4.5–2.5, the capacity N_a is reduced by half.

The frequency reuse efficiency K defined in [10] can be found from the inverse of the maximum points of Fig. 4. For example, at $\gamma = 4$, $K = 1.6$, which is higher than $K = 1.33$ for the reverse link [10], but still better than $K = 7$ for an analog FDMA system with $(C/I)_{\text{req}} = 18$ dB and $K = 3$ for a digital TDMA system with $(C/I)_{\text{req}} = 10$ dB, which are not using forward power control [3].

The distribution of G_2 in the home cell using the proper values of c and r_p is better shown in the three-dimensional (3-D) plots shown in Fig. 6. For $\gamma = 4.5$, shown in Fig. 6(a), the users in the cell are supported with little variation of the C/I compared to the rapid decrease away from the cell center for $\gamma = 2.5$, as shown in Fig. 6(b). The lowest level of the C/I or the "service hole" referred to in [7] can be observed within a region located between the cell boundary and about $0.65R$ from the cell center. It can also be seen from Fig. 6 that C/I is sensitive to θ_o only at $r > 0.65R$.

To show how the propagation environment can affect the estimate of capacity, $G_{2\min}$ is plotted in Fig. 7 versus the total number of interfering rings J for different values of γ. It is seen than with lower γ, a larger number of interfering rings impact the capacity N_a. For example, the capacities estimated with $J = 1$ are higher than those with by 8.5% and 43% for $\gamma = 3.5$ and $\gamma = 2.5$, respectively.

V. Conclusion

This paper shows that for an optimum distribution of the C/I and for highest capacity, forward power control should be applied with proper factors chosen according to the value of γ. With these factors the capacity is nearly doubled compared to the case without power control for a range of γ between 4.5–2.5. The region in the cell that experiences a relatively low level of C/I becomes wider as γ decreases. The forward CDMA cellular system capacity is strongly dependent on γ and is found to be reduced by about 50% as γ changes from 4.5 to 2.5. These re-

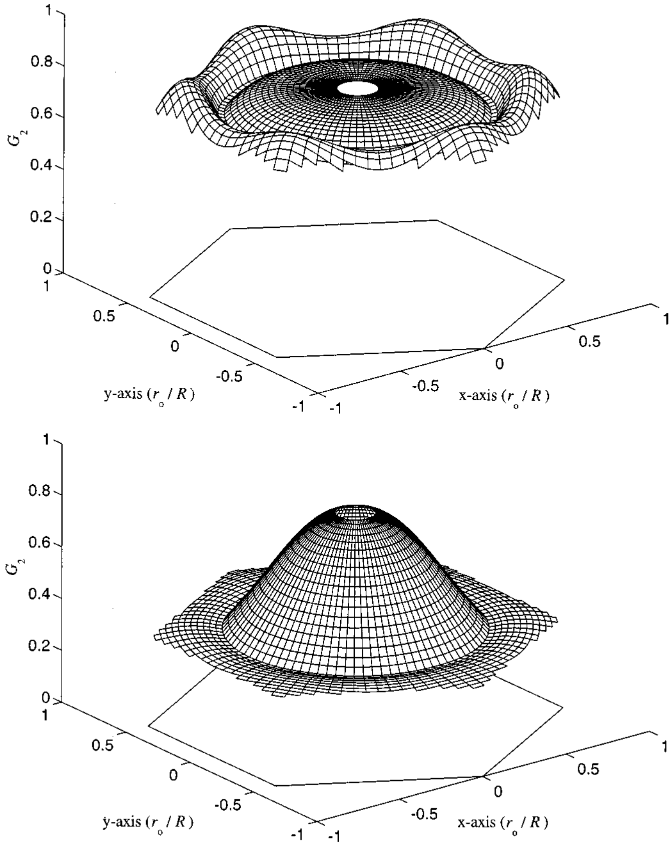

Fig. 6. Performance factor $G_2(r_o, \theta_o)$ using the proper power-control factors. $0.1 \le r_o <$ cell boundary, $0 < \theta_o \le 2\pi$. The hexagon determines the cell boundary in the x–y plane, where the x-axis is parallel to the i-axis in Fig. 1. (a) $\gamma = 4.5$. (b) $\gamma = 2.5$.

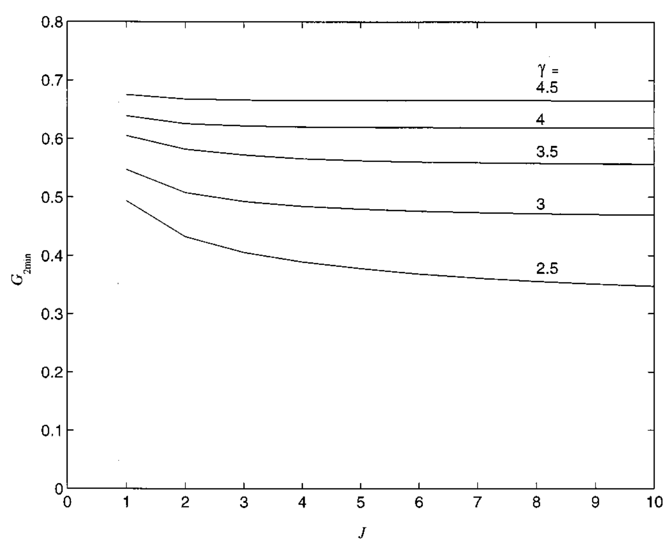

Fig. 7. Minimum of the performance factor G_2 using the proper power-control factors versus the total number of the interfering rings J.

sults are optimistic since they are based on an assumed path-loss law for propagation. The effects of shadowing and multipath fading are not included.

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REFERENCES

[1] W. C. Y. Lee, "Overview of cellular CDMA," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 291–302, May 1991.

[2] K. S. Gilhousen, I. M. Jacobs, R. Padovani, A. J. Viterbi, L. A. Weaver, and C. E. Weatly, "On the capacity of a cellular CDMA system," *IEEE Trans. Veh. Technol.*, pp. 303–311, May 1991.

[3] W. C. Y. Lee, *Mobile Cellular Telecommunications Systems*. New York: McGraw-Hill, 1989.

[4] ——, *Mobile Communications Design Fundamentals*. New York: Howard Sams, 1986.

[5] T. S. Rappaport and L. B. Milstein, "Effects of radio propagation path loss on DS-CDMA cellular frequency reuse efficiency for the reverse channel," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 231–242, Aug. 1992.

[6] A. O. Fapojuwo, "Radio capacity of direct sequence code division multiple access mobile radio systems," *Proc. Inst. Elect. Eng.*, pt. I, vol. 140, pp. 402–408, Oct. 1993.

[7] R. R. Gejji, "Forward-link-power control in CDMA cellular systems," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 532–536, Nov. 1992.

[8] C Wang and O. K. Tonguz, "Forward link power control for cellular CDMA networks," *Electron. Lett.*, vol. 2, pp. 1195–1197, June 1993.

[9] M. Zorzi, "Simplified forward-link power control in cellular CDMA," *IEEE Trans. Veh. Technol.*, vol. 43, pp. 1088–1093, Nov. 1994.

[10] K. I. Kim, "CDMA cellular engineering issues," *IEEE Trans. Veh. Technol.*, vol. 42, pp. 345–350, Aug. 1993.

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